Exercise 32

(a) Sketch the graph of $f(x) = \sqrt{6-x}$ by starting with the graph of $y = \sqrt{x}$ and using the transformations of Section 1.3.

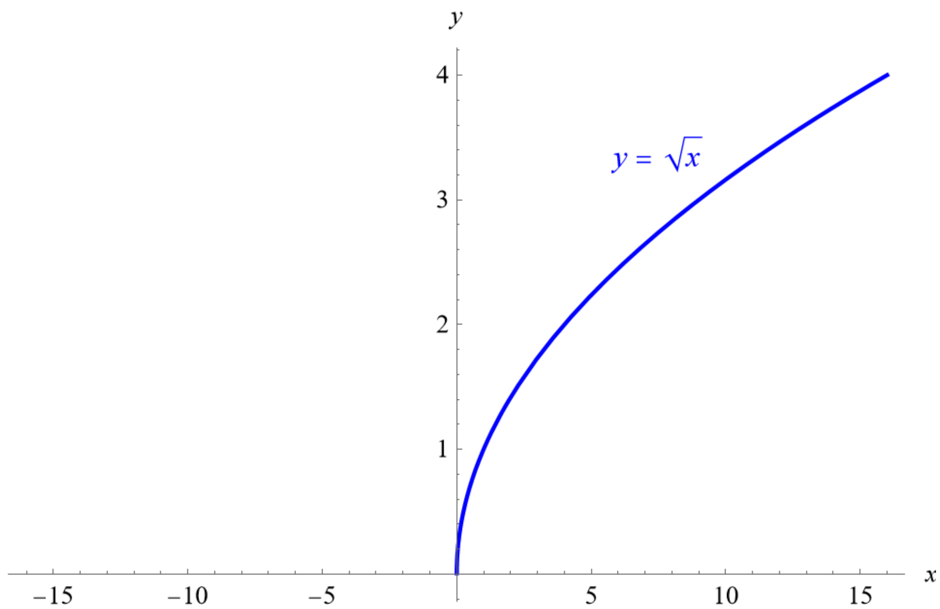
(b) Use the graph from part (a) to sketch the graph of f'.

(c) Use the definition of a derivative to find $f'(x)$. What are the domains of f and f'?

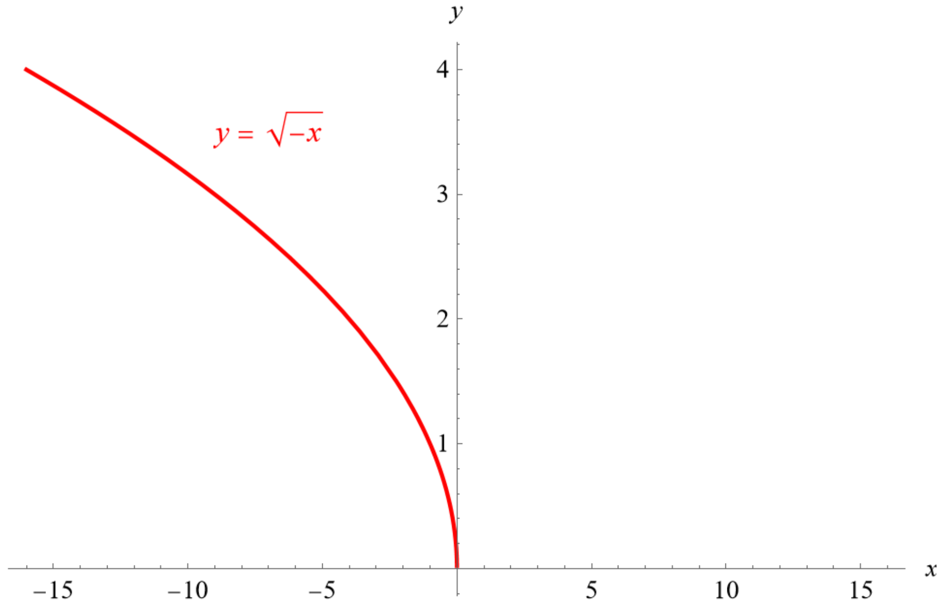
(d) Use a graphing device to graph f' and compare with your sketch in part (b).

Solution

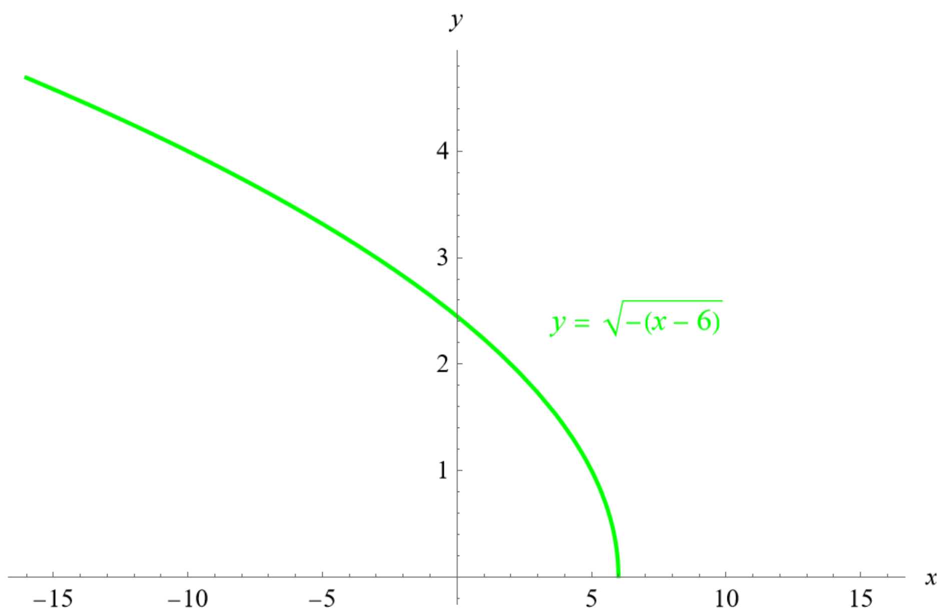
Rewrite the function as $f(x) = \sqrt{-(x-6)}$ and notice that the parent function is $\sqrt{x}$.



Replacing x with $-x$ reflects the graph over the y-axis.



Replacing x with $x - 6$ shifts the graph to the right by 6 units.



The domain of $f(x) = \sqrt{6 - x}$ is

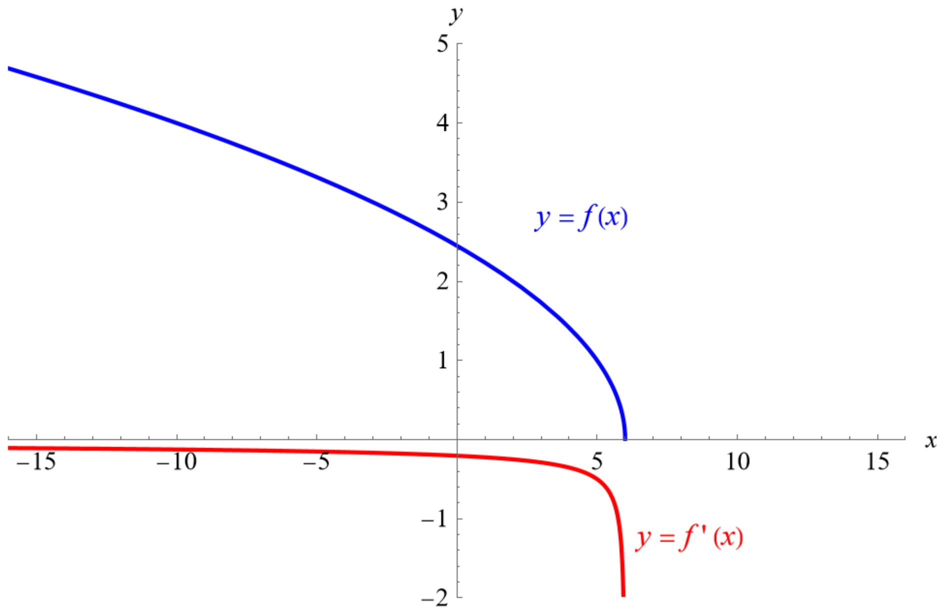
$$6 - x \geq 0$$

$$-x \geq -6$$

$$x \leq 6$$

$$\{x \mid x \leq 6\}.$$

Below is a graph of $f(x)$ and $f'(x)$ versus x.



Calculate the derivative of $f(x)$ using the definition.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{6-(x+h)} - \sqrt{6-x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{6-x-h} - \sqrt{6-x}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{6-x-h} - \sqrt{6-x}}{h} \cdot \frac{\sqrt{6-x-h} + \sqrt{6-x}}{\sqrt{6-x-h} + \sqrt{6-x}} \\
&= \lim_{h\to 0} \frac{(6-x-h) - (6-x)}{h\left(\sqrt{6-x-h} + \sqrt{6-x}\right)} \\
&= \lim_{h\to 0} \frac{-h}{h\left(\sqrt{6-x-h} + \sqrt{6-x}\right)} \\
&= \lim_{h\to 0} \frac{-1}{\sqrt{6-x-h} + \sqrt{6-x}} \\
&= \frac{-1}{\sqrt{6-x} + \sqrt{6-x}} \\
&= -\frac{1}{2\sqrt{6-x}}
\end{aligned}$$

The domain of $f'(x) = -\frac{1}{2\sqrt{6-x}}$ is

$$6 - x \geq 0 \quad \text{and} \quad 6 - x \neq 0$$

$$6 - x > 0$$

$$-x > -6$$

$$x < 6$$

$$\{x \mid x < 6\}.$$